

Don Patinkin's Ph.D. dissertation as the prehistory of disequilibrium theories

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This work is the outgrowth of ideas first presented in a doctoral dissertation submitted to the University of Chicago in 1947 and then further developed in a series of articles published in various journals and anthologies through the years 1948 to 1954. (1956: vii)

The opening sentence of *Money, Interest, and Prices*, has attracted the attention of most scholars who wrote about Don Patinkin's works in recent years. As shown by Boianovsky (2006), Merhling (2002) or Rubin (2002a), reading Patinkin's doctoral dissertation shed new light on his major work. However, these articles contain only partial presentations of the thesis. This essay contributes to fill in this gap. It offers a detailed presentation of the second part of Patinkin's Ph.D dissertation and claims that this document foreshadowed the research programs of disequilibrium theorists of the 1970's.

"On the consistency of economic models: a theory of involuntary unemployment", the text submitted to a committee composed of Gregg Lewis, Paul Douglas, Theodore O. Yntema and chaired by Jacob Marschak in August 1947, was composed of two parts. The first part analyzed how various general equilibrium models failed to incorporate money in a consistent manner. As indicated by the general title, the second part of the thesis, on involuntary unemployment, seemed the most important to the young economist. Actually, the two parts were written in reverse order. Patinkin started from the idea of explaining unemployment as the consequence of an "inconsistency" of the classical system (Patinkin, 1995, 379). But in a second phase, probably under the influence of the members of the Cowles Commission (Mehrling, 2002), he undertook to develop Lange's criticism of the traditional dichotomy of general equilibrium theory between the determination of relative prices and the determination of money prices.

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I will focus on the discussion of unemployment for it is the most surprising. The discussion on money was nothing but an early version of the first part of *Money, Interest and Prices*, though at this stage Patinkin did not realize that the real balance effect could cope for the inconsistency of the traditional dichotomy. The discussion of unemployment differed in many respects from the theory of unemployment finally presented in chapters 13 and 14 of *Money, Interest and Prices*. Patinkin adopted an unemployment equilibrium perspective (Rubin, 2002a) and rejected the Pigou effect (Rubin, 2005). But above all, he developed several concepts that were either abandoned or marginalized in the making of his book. In effect, as shown by Boianovsky in his careful study of the tortuous process leading from Patinkin's thesis to chapters 13 and 14 of *Money, Interest and Prices* "Patinkin's views about unemployment phenomena" did not evolved "linearly and cumulatively" (2006, 194).

The first part of the paper analyzes the research program underlying the thesis. The second part presents Patinkin's 1947 theory of involuntary unemployment. This part of the thesis displayed a mathematical apparatus composed of 325 equations and a burgeoning set of models. This apparatus has been simplified and its notations homogenised. Part 3 and part 4 study how Patinkin and the members of the Cowles Commission failed to realize the full implications of his research program. It is argued that with his "theory of compromise" and his concept of "additional restraint", Patinkin had gathered nearly all the ingredients to forge the twin notions of "spillover effect" and "dual decision hypothesis". The concept of "spillover" was developed later by Patinkin (1952, 1956). The "dual decision hypothesis" was put forward by Clower (1965). Both concepts were pieced together by Robert Barro and Herschel Grossman in a famous 1971 article and played a crucial role in disequilibrium macroeconomics. Part 5 concludes on Patinkin's position in the history of disequilibrium theories.

1. Patinkin's research program

The second part of Patinkin's thesis addressed three issues. First, Patinkin intended to derive a Keynesian macromodel from a Walrasian model. Second and in order to do so, he proposed to elaborate the microeconomics of action under "additional restraints". Finally, he wanted to reconcile the kind of adjustment at work in Samuelson's diagonal cross diagram with the law of supply and demand. I will show how this research program emerged out of the intellectual context of the mid-forties at the University of Chicago.

1.1. A Walrasian approach

During his first years as a student of the Department of Economics of the University of Chicago, from 1941 to 1943, Patinkin was barely exposed to the mathematical economics of Walras and the Lausanne school. The main reference with respect to price theory was Marshall, the only exception being the reading of Cassel (1932) who “reproduced Walras’ system without mentioning his name!” (Patinkin, 1995, 362). As a graduate student, between 1944 and 1945, Patinkin studied Marshall’s *Principles* for it was the basic text of the lectures given by Knight and Viner, the then dominant figures at Chicago University. He discovered mathematical economics only through the courses of Lange:

But Lange’s most valuable course for me was the one on Mathematical Economics (i.e. what was then called mathematical economics!). Here he systematically took us through the Mathematical Appendix of Hicks’ *Value and Capital* (1939), as well as Paul Samuelson’s path breaking article on “The stability of equilibrium” (1941), subsequently reproduced as chapter 9 of the latter’s *Foundations of Economic Analysis* (1947). My lecture notes from this course served me as a “reference volume” for many years to come. (1995, 372)

This initiation was continued by a course on mathematical economics and a seminar on Tinbergen’s 1939 econometric model of the United States both dispensed by Marschak. It was completed by Patinkin’s immersion in the context of the Cowles Commission, the organization in which he would write his doctoral dissertation.

Under the influence of Lange, Klein and the members of the Cowles Commission, Patinkin came to believe in the existence of an organic link between a Keynesian orientation and the practice of mathematical economics. His enthusiasm for mathematical economics at this stage is reflected in a manuscript probably written in 1948. In it, he stated “the people who are mathematical economists are mostly the young economists believing that government must play a larger role in the economic sphere”¹. State intervention required the knowledge of the parameters of the economic systems. This could be obtained by estimation of simultaneous equations systems. But the development of these systems implied mathematical economics.

A Walrasian orientation was conspicuous in the first part of the thesis on the integration of money in general equilibrium systems:

The first part consists of an examination of the “classical system”. As in the examination of any body of thought, the problem of textual interpretation immediately arises. To minimize this problem, I shall concern myself with the mathematical classical school of economics: Walras, Pareto, and certain of their followers. (1947, 5)

¹ See Merhling (2002) for a complete quotation.

But as we will show below, the models of the Lausanne school were also presented as the models of reference in the second part of the thesis. There Patinkin undertook a Walrasian reconstruction of Keynes' theory of unemployment in order to clarify its foundations.

The idea that an integration of the Keynesian and the Walrasian theories would strengthen the Keynesian position came from Lange (1938, 1944) and from Hicks (1939). In his autobiography, Patinkin (1995, 371) wrote that, in his course on business cycle theory, Lange offered a detailed presentation of the *General Theory* based on "The rate of interest and the optimum propensity to consume" (1938). In this article, Lange presented Keynes' apparatus, identified with the IS-LM model, as a simplified version of the Walrasian model:

Thus both the Keynesian and the traditional theory of interest are but two limiting cases of what may be regarded to be the general theory of interest. It is a feature of great historical interest that the essentials of this general theory are contained already in the work of Walras. (1938, 20)

And also:

Thus Mr. Keynes apparatus involves a considerable simplification of the theory. (1938, 23)

The basis for this interpretation was the belief that Keynes assumed a horizontal supply curve for labor (Lange, 1938, 31). This implied that labor market clearing was compatible with involuntary unemployment, so that a Keynesian unemployment equilibrium was only a sub-category of the Walrasian equilibrium. In *Price, Flexibility and Employment* (1944), Lange approached the *General Theory* from a different angle and argued that to prove the generality of Keynes' chapter 19 one had to restate its conclusions in a complete general equilibrium perspective. This approach was inspired by *Value and Capital*. In the introduction of his famous book Hicks announced that he would assess Keynes' results in the light of the "pure logical analysis of capitalism" in order to "[clear] up several important things [Keynes] left not very clear" (1946, 4). Retrospectively, this way of approaching Keynes' works seems far from being obvious. Keynes developed the *General Theory* from within the Marshallian tradition, a tradition that displayed a conception of the market more or less incompatible with the Walrasian conception². However, for Hicks (1934, 1939) and Lange (1932) Alfred Marshall and Léon Walras' conceptions of the market were complementary³.

For different reasons, Hicks and Lange did not probe into the relations between the IS-LM model and the temporary equilibrium model that both of them used to discuss Keynes' ideas. In 1939, Hicks did not consider IS-LM as a Walrasian construct. It was the model of the

² See Clower (1989) and De Vroey (1999a, 1999b and 2004).

³ On Lange's belief in the complementary between Marshall and Walras see Lendjel (2001).

General Theory, a framework which descended from the Marshallian tradition (Hicks, 1937, 150; 1950, 4). His own approach, developed in *Value and capital*, was different. Lange did write that IS-LM was Walrasian. But his interpretation of Keynes' unemployment equilibrium as a sub-case of the Walrasian equilibrium implied that the foundations of IS-LM were self-evident and need not be clarified. Finally, for both of them the important task was the reformulation of Keynes' main results in an authentic general equilibrium setting.

Unlike Hicks and Lange, Patinkin thought that the foundations of the macromodels inspired by the *General Theory* had to be clarified. Following his teacher, Lange, Patinkin believed that they could be derived from Walrasian models⁴. But, unlike him, he realized that in order to obtain a Keynesian model, one had to modify substantially the Walrasian framework. With hindsight, one may say that Patinkin put his finger on the microfoundation issue. He did so from two different angles: first, involuntary unemployment and, second, the inconsistency between Samuelson's representation of the Keynesian theory and the law of supply and demand.

1.2. From involuntary unemployment to the "additional restraint" concept

The need for microfoundations was one of the basic tenets of the Cowles Commission agenda under the directorship of Marschak⁵. This may explain how Patinkin tackled the problem of involuntary unemployment in his thesis.

Like many Keynesians involved in the neoclassical synthesis, Patinkin was a child of the Great Depression⁶. Hence, even though the choice of his thesis topic was not straightforward, it was not totally by chance that, in the end, he started to work on unemployment⁷. According

⁴ One can find an echo of Lange's interpretation in Patinkin's article on Keynes in the *New Palgrave*. There he contends that "a basic contribution of the *General Theory* is that it is in effect the first practical application of the Walrasian theory of general equilibrium" (1991, 27).

⁵ "The presentation of these reports was followed (or rather, constantly interrupted) by critical comments, a recurrent theme of which was the necessity (emphasized especially by Marschak) for basing the analysis –and the resulting empirical equations– on the principle of profit or utility maximization" (Patinkin, 1995, 385)

⁶ "On the other hand, the economic and social problems of the depression were part of our everyday experience. My father was in partnership with my uncle in a small business, which succumbed to serious financial difficulties. Fathers of close friends of mine had been unemployed for long periods, if not years. When we went downtown, we would see World War I veterans sitting at street corners behind up-ended empty fruit boxes in the bitter cold of Chicago winters, selling apples and pencils." (Patinkin, 1995, 360)

⁷ On this see Patinkin (1995, 378-9) and Merhling (2002).

to him, unemployment or more precisely involuntary unemployment was “the very question that brought forth the *General Theory*” (1949, 360). Yet, the concept that Keynes had put forward in chapter 2 of his book had not been given enough attention.

After the publication of the *General Theory* in 1936, a number of economists tried to capture Keynes message within formal models of the IS-LM type⁸. Among them, some, like Hicks (1937), completely left aside the concept of involuntary unemployment. The others accounted for it by assuming a horizontal supply curve of labor. The main advocate of this curve was Lange:

Involuntary unemployment in the Keynesian sense is not an excess supply of labor but an equilibrium position obtained by intersection of a demand and a supply curve, the supply curve of labor, however, being infinitely elastic over a wide range with respect to money wages, the point of intersection being to the left of the region where elasticity of supply of labor with respect to money wages becomes finite. (Lange, 1944, 6)

The notion of a “perfectly elastic supply of labor” can also be found in Franco Modigliani’s influential 1944 paper on IS-LM.

The way Patinkin himself tackled this issue was not based on Keynes’ definition of involuntary unemployment (Boianovsky, 2006, 209). His starting point was a critique of the way Lange and Modigliani defined and represented this concept, a critique that opened the way for a rational reconstruction. As he put it in a report to the Social Sciences Research Council, which financed his doctoral studies:

The point taken by my research was that none of the current theories of unemployment had emphasized its involuntary aspect. All of them were expressed in terms of demand and supply curve of labor, and workers were always ‘on their supply curve’. From my viewpoint this did not represent involuntary unemployment since workers were acting precisely as they desired, as represented by their supply curve. (Patinkin, Final report for the Social Sciences Research Council, June 1947)

Here, Patinkin clearly attacked the assumption of a horizontal supply of labor curve. The quotation shows that he assimilated the aggregate supply curve of labor and the curve expressing an individual workers’ desired supply of labor. Since the labor supply curve expressed the preferences of workers, their situation could hardly be called involuntary when trade was occurring at the intersection of the demand and the supply schedules, whether the latter was horizontal or not. In the conclusion of his dissertation, Patinkin made his point a bit differently by highlighting the absurdity of Lange and Modigliani’s approach. In their

⁸ See Darity and Young (1995).

reasoning, a slight change in the behaviors of workers could turn a situation of “involuntary unemployment” into a situation of full employment:

The artificiality of this definition is sufficiently demonstrated if one considers the case in which the supply curve instead of being horizontal in the interval $(0, N_1)$, the supply curve rises with a slope of 0.001! (1947, 115).

Instead of using Lange’s horizontal curve, Patinkin maintained the assumption of a standard “notional” supply of labor function and defined “involuntary unemployment” as a situation where workers were “off their supply curve”: “In order to be acting involuntarily, [workers] must be off their supply curve” (Final Report to the Social Sciences Research Council).

This approach raised a new problem. How could involuntary unemployment, so defined, match with choice theory? “Involuntary” in Patinkin’s “off the curve” sense meant “not chosen”. But economic theory only dealt with chosen outcomes. Economic action and voluntary action were one and the same thing. To escape this contradiction, Patinkin insisted on the relative nature of “involuntary action” and developed the concept of “additional restraint”. The fact that unemployed were not “on their curve” did not mean that they were on no curve. Though they did not achieve their Walrasian plans, they were still guided by a plan but one including “additional restraints”. Their behavior was then “involuntary” as compared to the behavior defined by Walrasian plans, plans without additional restraints, reflecting what agents “truly” desired”. Unemployed were “coerced” “in a relative sense only”:

Though the existence of widespread involuntary unemployment is historically irrefutable, economic theory has yet to deal adequately with it. Involuntary unemployment involves what might be called ‘relative coercion’: people cannot fulfill their desires as freely as under some other situation which serves as a norm of reference. Hence in order to give concreteness to the concept of coercion we must first define this norm of reference. It is theoretically meaning less to speak of involuntary unemployment without introducing a comparison between two alternative models: the actually existing one and some designated norm. I must emphasize that I define coercion and freedom in a relative sense only. People acting with the ‘normal’ freedom (i.e. under the restrictions to be found in the norm of reference) will (for the sake of brevity) be defined as fulfilling their desires freely. People acting under more than the ‘normal’ restrictions will be said to be coerced and prevented from fulfilling their desires. In most of what follows our norm of reference is defined as a model in which the economic unit is restricted only by the budget restraint and technological relationships (e.g. the production function). (1947a, 79)

The notion of “additional restraint” expressed with remarkable clarity the necessity of modifying the Walrasian budget constraint, the “budget restraint” in Patinkin’s language, in order to obtain a disequilibrium model out of a Walrasian framework. In the conclusion of his

thesis, Patinkin clarified further his thinking and deepened his insight in presenting a mathematical formulation of it⁹. He wrote down a maximization program showing how, in a disequilibrium state, agents' calculus would produce new behavior functions, the g_i functions, resulting from the combination of a Walrasian budget constraint and an additional constraint. These would express simultaneously the fact that agents suffered from rationing and the fact that they still had a margin of choice concerning their desired quantities. In this particular instance, Patinkin imagined "some sort of rationing control". This illustration referred to the context of World War II, when consumers had to face price controls and rationing. Not being able to purchase the desired quantities of some goods, the consumer would have to allocate differently his income taking "rationing control" into account. In the ensuing comments, Patinkin explained that the same approach could be applied to firms and unemployed workers. But if we do apply his approach in the case of unemployed, the g_i functions become "effective demands" à la Clower (1965) as opposed to the "notional" or Walrasian demands noted h_i resulting from the budget constraint only:

In general, for any given set of prices and income, the amount purchased, as given by g_i , will differ from that given by h_i . So even though the consumer may be on his g_i function, he is not fulfilling his desires; for the desires of the individual are defined as being represented by his f_i [sic] functions.

In a similar fashion, application of an additional restriction to the maximization procedures of firms and workers may still leave them free to maximize utility or profit. In this case, too, even though the worker is on a curve (of the type (62.7)), he is not on the one he "really" desires. [...]. Therefore we can still distinguish between N_r^S and N_r^S . (1947, 118)

All these elements show that by 1947 Patinkin had realized that to account for involuntary unemployment one had to modify the standard theory of choice and that the main modification concerned agents' constraints. Yet, all we have at this stage is the ideal-type of a theory of choice in disequilibrium. Patinkin still had to give an operational content to his general insight. He had to apply it to the behavior of firms and consumers in disequilibrium and see if it could help him elaborate a Keynesian model.¹⁰

⁹ See the appendix.

¹⁰ As shown by Boianovsky (2006, 206-8), another source of Patinkin's notion of additional restraint may have been his dissatisfaction with the mathematical representation of the formation of general equilibrium in Lange (1944) and Samuelson (1941) expressed in a 1946 manuscript. At this stage Patinkin ignored tâtonnement and its recontracting assumption. For him, whenever there was a gap between supply and demand some agents had to be "off their curve", a fact that was not captured by Samuelson's differential equations.

1.3. The diagonal cross diagram with flexible prices

As an historian of economic thought, later in his career, Patinkin identified the equilibrating effect of output variations as the “major novel feature of the *General Theory* and its central message” (1982, 11). This point was illustrated by Samuelson’s 1939 diagonal cross diagram. Patinkin also made extensive use of this Samuelsonian representation of the goods market in *Money, Interest and Prices*. The reader of the thesis realizes that he interpreted Samuelson’s theory of income determination as the core of the Keynesian theory as early as 1947. This conception probably derived from Lange’s teaching and a careful reading of Samuelson’s 1941 paper on the “Stability of Equilibrium”. In this text, Samuelson analyzed the stability of an IS-LM model where the rate of interest and income were the adjusting variables. The price level and its variations were totally absent from this representation of the Keynesian theory. The foundations of such a model were not obvious. Two approaches were possible. One could assume that wages and prices were both fixed, the option that Hansen (1953) would choose. The models of Hicks (1937) and Modigliani (1944) offered a second option. They presented money income as the variable adjusting savings and investment. But in their models, money income adjusted only because price variations modified the labor demand and firms’ output (at least up to full employment where pure inflation became the only cause of the increase in money income). In other words, nominal wages being rigid, the level of prices was the underlying variable adjusting aggregate supply and aggregate demand¹¹.

But another key aspect of Patinkin’s interpretation of the Keynesian theory was his rejection of the assumption of wage rigidity and, a fortiori, price rigidity. This point is not discussed explicitly in the thesis. The reader can see only that such assumptions are nowhere introduced. Patinkin’s correspondence clarifies his position.

In “Liquidity Preference and the Theory of Money and Interest” (1944) Franco Modigliani offered an alternative to the interpretation of the Keynesian theory based on the liquidity trap argued by Hicks (1937) and by Lange (1938). He maintained that the characteristic assumption of Keynes’ theory was the assumption of downward nominal wage rigidity:

It is usually considered as one of the most important achievements of the Keynesian theory that it explains the consistency of economic equilibrium with the presence of involuntary unemployment. It is, however, not sufficiently recognized that, except in a limiting case to be considered later, this result is due entirely to the assumption of ‘rigid wages’ and not to the Keynesian liquidity preference. (1944, 65)

¹¹ For a similar interpretation see Dutt (2002, 332).

This approach laid down the standard meaning of the adjective “Keynesian”. Yet, it did not convince Patinkin. The reason was stated very clearly in a letter written in April of 1948 to Franco Modigliani:

Now, if the whole purpose of Keynes is to say that with rigid wages we can have unemployment ‘equilibrium’, I really do not see his contribution. This is a point that would have been admitted by classical economists themselves, but Keynes (on page 12, line 9 and following) seems to argue that the classical position on this point was wrong : that there could be unemployment for other reasons. (Patinkin, 1948, 2)

According to Patinkin, the explanation of unemployment through wage rigidity was the classical one. If this explanation was ascribed to Keynes, then the *General Theory* brought nothing new. Therefore, a Keynesian theory had to demonstrate that unemployment was not due to wage rigidity but to ‘other reasons’.

Patinkin found these reasons in the work of Klein. He wrote, in his “Training of an Economist”, that during his stay at the Cowles Commission “there were stimulating discussions with Lawrence Klein on the manuscript of his then forthcoming *Keynesian Revolution* (1947)” (1995, 385). In this book, Klein argued that the characteristic assumptions of the Keynesian model were that investment and savings were “interest inelastic” or “insensitive” in respect to variations of the rate of interest. Given these assumptions, he asserted that an IS-LM model with flexible prices and wages could have no equilibrium solution. The problem, in this setting, came from the fact that equilibrium of savings and investment for a full employment income could imply a negative rate of interest. Yet such a negative value was excluded. As a result, the aggregate demand remained less than the aggregate supply of goods. Klein interpreted this result as a demonstration of the incapacity of the market system to guarantee a perfect coordination of economic activities. But such a situation described a “hyper-deflationary” system and was not realistic. Therefore, he introduced wage rigidity in his model to guarantee the existence of an unemployment equilibrium. Still, the exogenous wage was not the cause of unemployment in his model: “In the Keynesian system lower wages need not do any good” (Klein, 1947, 87). Given the psychology of households and entrepreneurs, there was no equilibrium of savings and investment compatible with full employment. Hence, even with the exogenous wage reduced to zero, full employment would not be restored.

Klein was a major source of inspiration for Patinkin’s thesis. But while he adopted his explanation of involuntary unemployment, he dismissed his Keynesian version of IS-LM. Patinkin actually jumped from the idea that involuntary unemployment was not a consequence of money-wage rigidity to the contention that wage rigidity, and *a fortiori* price

rigidity, had to be excluded from the Keynesian theory. In other words, this theory had to demonstrate that a perfectly competitive system with price and wage flexibility could suffer from chronic unemployment. His correspondence offers clear evidence in support of this interpretation, as the following passage taken from a letter to Wassily Leontief illustrates:

The second issue, and one with which, as I understand it, Keynes is really concerned is: Is a wage decrease the way to solve the problem of unemployment? The Classical answer is yes; Keynes' answer is no for reasons which he sets out in chapter 19 of the General Theory. (...). This position can be maintained without assuming wage rigidity. In fact, what it says is that despite wage flexibility, full employment will not be restored. (Patinkin to Leontief, 21 February 1950).

This position was probably inspired by Lange. His 1944 book on *Price Flexibility and Employment* focused on chapter 19 of the *General Theory* and reinterpreted Keynes' question as a stability issue. It started from a situation in which the economic system was in excess supply on the labour market and studied whether the adjustment of wages and prices would take it to a general equilibrium state. In this perspective, the Keynesian message did not depend on an assumed rigidity of wages. Evidence of Lange's influence on this point can be found in the notes taken by Don Patinkin during his courses. For instance, in Patinkin's notebook from the course on business cycle theory, one can read: "So in Keynesian system for permanent unemployment *don't need rigid wages*" (Business Cycle Theory, Summer 1945: 96, Patinkin's emphasis).

To conclude, when Patinkin wrote his thesis, his representation of the Keynesian theory was Samuelson's diagonal cross diagram combined with flexible prices and flexible wages. But Samuelson's interpretation of the effective demand principle, featuring income as the adjusting variable, was at variance with the law of supply and demand, a principle reigning over Lange's 1944 analysis. How could one reconcile both adjustment mechanisms? How could one articulate the diagonal cross diagram and the general equilibrium theory without assuming wage or price rigidity?

2. The macroeconomics of "inconsistent systems"

Starting from the preceding ingredients, the Walrasian perspective, the problem of involuntary unemployment and the ambition to find something better than IS-LM with rigid wages, Patinkin developed a bold construction. His theory would show that when the price mechanism did not lead the economic system towards general equilibrium in the Walrasian meaning of the terms, a distinct kind of equilibrium would emerge out of a bargaining

process. This competing general equilibrium concept would feature simultaneously involuntary unemployment and flexible prices.

2.1. Microfoundations of the classical macromodel and origins of unemployment

In the introduction of the second part of his thesis, on involuntary unemployment, Patinkin stated that his “first task [would] be the formation (by aggregation) of these macromodels from the Casselian equations previously considered” (1947, 51) ¹². In other words, Patinkin’s intended to derive his macromodels from a Walrasian one. To begin with, he explained how to derive individual demand functions including money income as one of their components¹³. Then he contended that his analysis offered the basis to obtain a “Casselian system of equations”, a set of n equilibrium conditions of the type supply of commodity i equals demand. The aggregation of the supply and demand functions of this system was then supposed to give rise to the following classical macromodel:

$$X^D = G(Y, r) \tag{1}$$

$$X^S = \eta \tag{2}$$

$$X^D = X^S \tag{3}$$

$$Y = X^S \tag{4}$$

$$B(r, p, Y) = 0 \tag{5}$$

$$M(r, p, Y) = 0 \tag{6}$$

The function X^D defined the aggregate demand for goods. It increased in respect to income and decreased in respect to the rate of interest. Patinkin ignored the “Pigou effect” for theoretical as well as empirical reasons¹⁴. The aggregate supply of goods was constant. It represented the full employment supply derived from the labor market equilibrium. Eventually, the aggregate income was defined by the aggregate supply. Equations (5) and (6)

¹² Or “My interest is in moving from the semi-microequations to the macroequations which play so prominent a role in current economic discussion” (Patinkin, 1947, 57).

¹³ More on this point below.

¹⁴ Patinkin justified this neglect stating that “whatever empirical evidence does exist lends little support to [Pigou’s] hypothesis” (1947, 73). For the theoretical reason and Patinkin’s evolution on this point see Rubin (2005).

are respectively the condition for equilibrium on the market for bonds and the condition for equilibrium on the market for money.

Following Klein (1947), Patinkin intended to show that this classical system could be “inconsistent” or that it could have no equilibrium solution:

The argument is made that the cause of this inconsistency is the insensitivity of savings and investment to fluctuations in the interest rate. This insensitivity creates a situation in which there exists (potentially) more savings at the full employment level than can be offset by investments. Hence this full employment level cannot be brought into existence. (Patinkin, 1947, 51)

In terms of the above system, this amounted to say that the equilibrium condition on the goods market $\eta = G(\eta, r)$ might imply a negative rate of interest¹⁵. Such a rate was deemed incompatible with equation (6) for it would have entailed an infinite demand for money. In other words, the equilibrium condition on the market for money implied the condition $r \geq 0$.

By definition, in a system without full employment solution, price and wage flexibility could not bring back full employment. In his dissertation, Patinkin interpreted this result as demonstrating that the problem of involuntary unemployment was inherent to the classical system, which represented a perfectly competitive market economy:

If the worker is unemployed, it is not because he wants to be, but because the basic inconsistency of the system makes it impossible to satisfy his desires. (Patinkin, 1947, 116)

This was only the first stage in the elaboration of Patinkin’s theory of involuntary unemployment.

2.2. Unemployment equilibrium: the “macroeconomic” approach

For Patinkin, a model without an equilibrium solution could “not describe the real world” (1947: 78). Therefore, the classical model had to be amended to obtain an unemployment equilibrium. But he refused to resort to the assumption of wage rigidity and put forward instead what he coined as a “theory of compromise”.

a) The determination of employment on the market for goods

In order to develop his unemployment theory, Patinkin started from a simplified classical model. The economist still claimed to deal with a monetary economy but eliminated the rate

¹⁵ Note that Patinkin, like Klein (1947), did not distinguish between the money rate and the real rate of interest and implicitly assumed that the expected rate of inflation was nil.

of interest from the demand for goods¹⁶. Variations of monetary aggregates did not affect the markets for goods and labor so the equations for the bonds and the money markets could be put aside. The system consisted of equations (2), (3), (4) and (7):

$$X^D = X^D(Y) \quad (7)$$

Patinkin used Samuelson's diagonal-cross diagram to illustrate his theory. From this point of view, one defect of Keynes' theory was that it ignored the global supply of goods as derived from the equilibrium on the labor market (1947, 87). Once this supply side was reintroduced in the model, the inconsistency pointed earlier could appear clearly. Supply, corresponding to the equation $X^S = \eta$, was represented by a vertical line on the diagram:

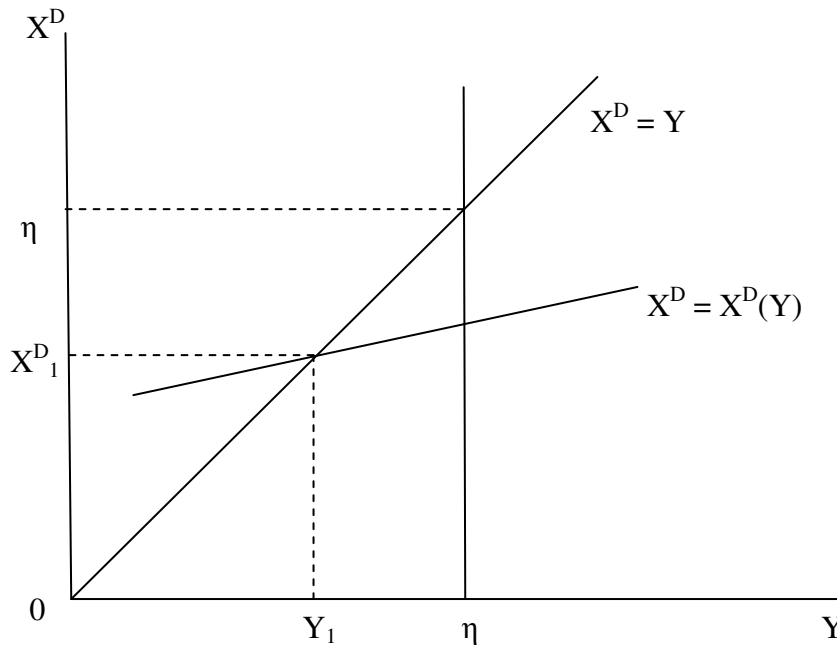


Figure 1 (Patinkin, 1947, 83)

The fact that the supply curve and the demand curve did not cut the bisecting line on the same point illustrated the absence of equilibrium.

According to Patinkin the initial “inconsistent system” determined two distinct values for income. The first one, η , was defined by the labor market clearing or by the “subsystem” (2), (3) and (4). It expressed the income “desired” by the suppliers of goods. The second one,

¹⁶ “Let us now consider the real part of the system and see what happens when it becomes overdetermined. For this purpose I assume that r does not enter the expenditure function.” (Patinkin, 1947, 82)

defined by the equation $Y = F(Y)$ or by the “subsystem” (7), (3) and (4), was the income Y_I on figure 1. This value expressed the income “desired” by the buyers of goods. In order to account for the existence of an equilibrium with unemployment, Patinkin introduced his “theory of compromise”. He explained that when such “disagreement” occurred between the two sides of the market, a “compromise” had to be reached the outcome of which depended on the respective “bargaining powers” of suppliers and demanders¹⁷. The stronger the “bargaining power” of demanders, the nearer to their desired income they would get. Thus, a priori, the equilibrium income could take any value between the full employment income and the income determined by the outlet constraint. Happily enough, according to Patinkin, in the “normal situation” and in a case of excess supply, the demanders were the one to lay down their conditions:

Assume, for example, that suppliers have no bargaining power. This means that buyers need make no compromise and can force the suppliers to accept the level of Y which they (the buyers) desire. (...). I believe this to be a description of the normal situation in our economy. It should be emphasized that this superior bargaining power implies no personal relationship between buyers and sellers. It is reflected by the simple institutional fact that as a rule buyers can obtain anything they want, and cannot be forced to purchase anything they do not; suppliers, on the other hand, are not always free to sell the entire amount they desire. They are frequently restricted by a “limited market”. (1947, 84)

A measure of unemployment could then be given by the difference between η and Y_I . η being considered as the « norm of reference » or the full employment income.

It was at this stage of the thesis that Patinkin touched upon the fundamental issue raised by Samuelson’s diagram. How did it square with the standard price mechanism? The introduction of an aggregate supply curve into the diagram revealed the problem¹⁸. What appeared as an equilibrium in the standard Keynesian presentation became a disequilibrium situation:

The failure of Keynes and his followers to discuss the issue in terms of overdeterminacy was due to the fact that they almost never introduced the supply curve explicitly into the discussion. (1947, 87)

According to Patinkin, the Keynesian adjustment mechanism was justified because the law of supply and demand failed to clear the markets. This mechanism was now founded on his

¹⁷ This approach was also inspired by Klein (1947, 87) who referred to the “superior bargaining power of the employer over the employee” to explain why “workers will not remain on their supply curve of labor”.

¹⁸ For a detailed analysis of Patinkin’s contribution to the theory of aggregate supply see Boianovsky (2002).

theory of compromise. Though on the goods market and in a “normal situation” this boiled down to a short side condition.

b) The determination of the real wage on the labor market

Patinkin presented the preceding model in chapter 7 of the thesis. He analyzed the situation that this model implied on the labor market in chapter 9. Whereas chapter 7 presented the compromise between the group of buyers and the group of sellers, chapter 9 was about the compromise between firms and workers¹⁹. The model finally consisted of equations (2), (3), (4), (7) plus the following equations:

$$Y = \Phi(N) \tag{8}$$

$$(W/P)_I = \Phi'(N) \tag{9}$$

$$(W/P)_2 = \Pi(N) \tag{10}$$

$$W/P = \gamma \Pi(N) + (1 - \gamma) \Phi'(N) \tag{11}$$

Once output was determined, equations (9), (10) and (11) determined the equilibrium real wage. The “theory of compromise” was again at work. In a “normal” economic situation, income was fixed by the demand for goods. To this income corresponded a certain level of employment given by the production function. There were two different real wages in these conditions. The real wage defined by the labor demand curve (equation 9) was the maximum available to workers hence the one they desired. Inversely, the real wage defined by the labor supply equation was the minimum that workers would accept hence the one desired by firms. This disagreement called for a compromise. As in the preceding case, the outcome depended on the “bargaining powers” of both parties. Equation (11) summarizes the formalization proposed by Patinkin. The parameter γ indicated firms’ “bargaining power”. The bigger it was the nearer firms got to the minimum wage. But, if an agreement was reached, γ also indicated the “bargaining power” of workers. If γ was small the workers got a real wage near the one they desire. This key parameter determined the equilibrium real wage like a weighted average of the wages desired by the two sides of the market. This analysis was illustrated by figure 2:

¹⁹ In chapters 6 and 7, Patinkin actually referred to an economy of “individuals” producers and consumers alike whereas in chapter 8 and 9 he distinguished between, firms and workers. But in spite of this discontinuity in the verbal presentation, formally, the model of chapter 9 was only an extension of the models of chapter 6 and 7.

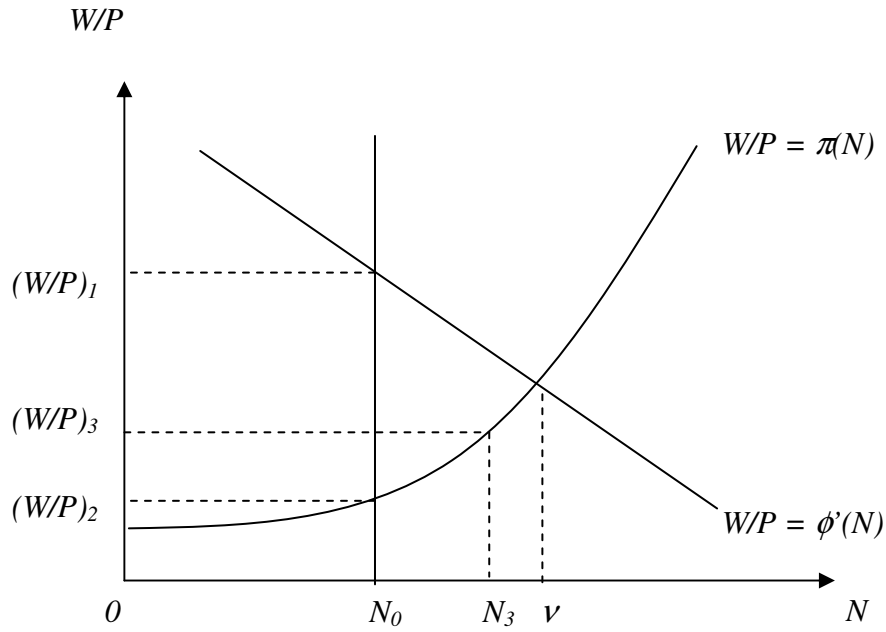


Figure 2 (Patinkin, 1947, 102)

On the basis of this diagram, Patinkin examined whether the best measure of unemployment was $U = N_3 - N_0$ or $U = \nu - N_0$. The former was preferred because it carried with it the notion of involuntary action. After that, Patinkin analyzed the effect of a change in the bargaining power γ on employment and the real wage. He showed that in a system where the propensity to spend of workers was higher than the propensity to spend of employers, a rise of workers bargaining power would increase both the real wage and the employment level.

2.3. Unemployment equilibrium: the “microeconomic” approach

In chapter 8 of his Ph.D. thesis, entitled “Unemployment and inconsistent systems: microeconomics”, Patinkin undertook to develop the microeconomic counterpart of his macroeconomic model²⁰. The approach was microeconomic in a double meaning. First, the model was multisectoral. Second, Patinkin detailed agents’ maximization programs. The purpose of the multisectoral structure was to show how unemployment was spread among

²⁰ “Our attention will be concentrated on the microsystem lying behind this [macroeconomic] system” (1947, 88).

industries and workers²¹. Since Patinkin's results on this matter are not compelling²², we will not alter very much the substance of his reasoning by dealing with an aggregate version of his model. The clarification of agent's maximization program served another aim. Patinkin wanted to show the role played by his concept of "additional restraint" in his theory of involuntary unemployment:

I must be clear at the outset that the present chapter [chapter 8] is not intended to be a description of the real world. Its primary purpose is to be a rigorous exercise in the use of the concepts introduced in section 38 [the section defining involuntary action as action under additional restraint]. Through this chapter I hope (a) to define precisely the meaning of action under additional restraints, (b) to emphasize the relativity of the concept of unemployment to the norm by which it is measured, and (c) to introduce explicitly the role of bargaining power in the economic system. (1947, 88).

Last but not least, the economist believed that it was "more meaningful to have the compromise between employers and workers" (1947, 88). Therefore, in contrast with his "macroeconomic" model, the "microeconomic" model focused exclusively on the labor market.

The classical model used as a benchmark in chapter 8 shared the simplifications introduced to deal with the Keynesian macroeconomic model. But the equilibrium condition for the labor market was now explicit. The model consisted of equations (3), (4), (7), (12) and (13).

$$Y = F(N) \tag{12}$$

$$N^S(W/P) = N^D(W/P) = N \tag{13}$$

At this stage of his presentation, Patinkin did not mention the money market. But we can assume that he implicitly put it aside owing to Walras' law. This being said, his system was clearly "inconsistent". Save a happy coincidence, equations (3), (4), (7) and the subsystem (12)-(13) would determine two different values for real income and the system would have no solution. There remained to analyze the functioning of the system in disequilibrium. In Patinkin's language, one had to elaborate a theory of compromise.

Given Patinkin's approach, the definition of the disequilibrium configuration of his system was a difficult task. In order to determine agents' notional demands, Patinkin may have started from an arbitrary price vector. In other words in might have imagined some kind of

²¹ "The system is incomplete until we show how this unemployment is spread among the different workers" (Patinkin, 1947, 88).

²² The repartition of unemployed between sectors was determined by exogenous parameters.

non-tâtonnement process. But the notion of tâtonnement was absent from the thesis. And probably because he refused to assume price rigidity, Patinkin found a different way out. He solved the difficulty by using a procedure similar to the one he employed in his “macroeconomic” analysis. He built two subsystems out of his classical model, each one now being supposed to determine a real wage and an employment level. The idea was the following. The Walrasian model was overdetermined, it contained one equation too many. Hence, Patinkin abandoned the equilibrium condition for the labor market. Then he considered two cases. In the first one, he assumed that “workers direct the economy” (1947, 91). The latter calculated a real wage that would equilibrate the market for goods assuming that production was a function of the labor supply. In the second one, “employers control the economy” (1947, 92). Like workers, firms calculated a real wage that would equilibrate the market for goods assuming this time that production was a function of labor demand. The first sub-system, called “workers’ model”, determined a level of employment noted N^e considered by Patinkin as the level desired by workers or the “full employment level”. The second sub-system, called “employers’ model”, determined N^f , considered by Patinkin as the level of employment desired by firms. The comparison of both magnitudes was supposed to measure the degree of “inconsistency” of the classical system: “In general, N^f will be different from N^e ” (Patinkin, 1947: 92). In practice Patinkin assumed that $N^f < N^e$. With hindsight, the configuration studied by Patinkin was one of equilibrium on the goods market, excess supply on the labor market and excess demand on the money market.

I can use my simplified version of his model to illustrate Patinkin’s approach. But this immediately reveals a key ambiguity of his presentation. In the introduction of chapter 8, Patinkin wrote explicitly that agents’ demands for goods were supposed to be the Walrasian demands:

I will assume that no matter what happens consumers will be able to fulfill their desires, and will not have to compromise. This means that the second step in the maximization of utility (cf. above, section 28) proceeds unhindered and need not be considered in microeconomic detail. Consequently we can assume that individuals as consumers have already maximized utility to yield the demand functions for the product of each industry. (1947, 89)

Walrasian demands for goods should have been functions of the relative prices of the system and in particular of the real wage. But when Patinkin wrote down the equations defining the demands for goods in chapters 6, 7 and 8, he presented them as functions of Y or N , the effective levels of output and employment, just like our equation (7). In other words, Patinkin did not distinguish between Walrasian demands and standard Keynesian demand functions. If

we interpret his aggregate demand function as a Keynesian one, the following results obtain. To begin with, equation (13) was put aside. Then, in the “workers’ model”, the labor supply function was introduced in the production function and the production function replaced income in the equilibrium condition for the goods market. The result was an equation giving an equilibrium real wage (equation 14) and, consequently, the levels of employment and production (equations 15 and 16):

$$F [N^S(W/P)] = X^D \{F [N^S(W/P)]\} \quad (14)$$

$$N^e = N^S(W/P) \quad (15)$$

$$Y = F (N^S) \quad (16)$$

In order to obtain the “employers’ model”, Patinkin used the same procedure with labor demand replacing labor supply:

$$F [N^D(W/P)] = X^D \{F [N^D(W/P)]\} \quad (17)$$

$$N^f = N^D(W/P) \quad (18)$$

$$Y = F(N^D) \quad (19)$$

Retrospectively, one realizes that, interpreted in this way, Patinkin’s approach would have suffered from a serious flaw. The way he presented them did not modify the fact that the adjustment variable of equations (14) and (17) was still real income Y . As a consequence, the two sub-systems would determine the same income and the same level of employment, Y_I and N_0 , the solutions of the Keynesian macroeconomic model. The supply price of labor and the demand price did differ but the Patinkin’s procedure failed to measure unemployment defined as a gap between N_e and N_f . A second interpretation is possible though. If we take into account Patinkin’s intention and forget the way he wrote down his demand functions, his procedure proves to be valid. Once equation (7) is replaced by a function of the real wage, the “workers’ model” and the “employers’ model” can give rise to two distinct levels of employment with $N^e < N^f$.

After having characterized the situation of the economy by measuring the gap between N^e and N^f , Patinkin used his “theory of compromise” to define the transactions agents realized on

the labor market²³. The level of transactions was determined by the respective bargaining powers of agents:

$$N^D = N^f + \alpha(N^e - N^f) \quad (20)$$

$$N^S = N^e + \beta(N^f - N^e) \quad (21)$$

$$N^D = N^S \text{ if, and only if } \alpha + \beta = 1 \quad (22)$$

As indicated by equation (20) firms would employ a quantity of labor equal to $\alpha(N^e - N^f)$ in addition to their desired labor demand N^f . Patinkin called α the “compromise coefficients” of firms. The greater this coefficient the more firms “took into account the overall desires” of the workers group (1947: 93). In the same way, the quantity of labor supplied by workers was a function of their desired labor supply and of their “coefficient of compromise” β . Equation (22) defined the equilibrium condition for transactions: it showed that “for the successful functioning of the economy, it is necessary that the sum of compromise coefficients cover the difference submitted to compromise” (1947, 93).

Since transactions determined by the “theory of compromise” did not coincide with the transactions desired by agents, they implied the existence of ex-post constraints that had to be introduced in agents’ decision programs:

Assume that a successful compromise is reached. We can now consider [(20)] and [(21)] as additional restrictions on the maximization procedure of firms and workers. (1947, 93)

Firms maximized the following objective function:

$$PY - WN^D - \lambda_1[Y - F(N)] - \lambda_2[N^D - N^f + \alpha(N^e - N^f)]. \quad (23)$$

And workers maximized :

$$U(WN/P, L) - \mu_1[N^S + L - T] + \mu_2[N^S - N^e + \beta(N^f - N^e)] \quad (24)$$

Patinkin decomposed households’ choice in two stages. In the first stage they chose between “real income” WN/P and leisure L given a constant amount of time T . In the second stage only they decided how to “allocate their income among different purchases” (1947, 53).

²³ “The model Q_1 and Q_2 are hypothetical models whose primary purpose is to define the constants $[N^D]$, $[N^S]$, N^e , and N^f . But they do not attempt to “describe the real world;” for this purpose we would have to construct a third consistent model, Q_3 ” (1947, 92)

The fact that the “additional restraint” imposed by the compromise on the labor market impinged on consumers choice and implied an effective demand for goods distinct from the notional demand was not seen by Patinkin. Writing down agents’ maximizing program was important mainly because it showed that they acted “involuntarily”:

In the terminology of section 38, these additional restraints (i.e., additional to the budget identity and production function) mean that firms and workers are being coerced. They are being forced to compromise, and prevented from fulfilling their desires. (1947, 94)

Once the level of employment was determined, and the involuntary nature of actions clarified, output, the demand for goods and the real wage were supposed to be determined respectively by the production function, the notional demand function for goods and the equilibrium condition on the market for goods. Here, the ambiguity pointed out above resurfaced. In Patinkin’s Keynesian model only one level of employment was compatible with the condition $Y = X^D(Y)$ and nothing assured that the employment level resulting from the bargaining procedure would be compatible with equilibrium on the goods market. If Patinkin had seen this he might have imagined a second round of negotiation on the market for goods, a negotiation that would have settled the level of employment.

Once he had presented the general structure of his model, Patinkin elaborated upon his theory of compromise. Each agent was endowed with a “compromise coefficient”. α_{ij} was the coefficient of firm i in industry j and β_{ijk} the coefficient of worker k in firm i of industry j . On this basis he discussed the role of social conflict in the attainment of equilibrium. The sum of coefficients α and β might not be equal to one. This meant that firms and workers failed to compromise and that the system “ceases to function” (1947, 97):

Then the individuals might attempt to influence the α_{ij} and the β_{ijk} of the others to make them more willing to compromise. Thus resort may be had to strikes or lockouts, violent or peaceful. Or one group may succeed in getting Congress to pass laws in its favour (e.g. laws for or against collective bargaining, unions, etc.). (...). In extreme cases one of the groups may resort to revolution to force the others to compromise. The whole system then breaks down until the new realignment of powers is settled. (1947, 97)

Compromise coefficients were supposed to reflect “institutional factors”. Younger workers and unorganized workers would have higher compromise coefficient than older workers and members of unions and would have to accept more unemployment. But a conflict might also appear between the workers themselves about the distribution of employment. Patinkin also presented a model in which the compromise was being determined at the industry level.

Although Patinkin devoted a lot of attention to the careful exposition of his theory of compromise, this attempt to graft conflict and class struggle onto a Walrasian framework did not convince the members of the Cowles Commission.

3. Appraising the thesis: Cowles Commission members on the theory of compromise

The reports made on the drafts of the thesis by the members of the Cowles Commission criticized heavily his theory of unemployment. Jacob Marschak, the chairman of his thesis committee, Tjalling Koopmans, then a Research associate, Herbert Simon, who had just been engaged as a Study Co-Director, and Hans Neisser from the New School, who had been asked for comments by Marschak, all pointed to the shortcomings of Patinkin's "theory of compromise".

First, they criticized Patinkin's demonstration of the classical system's inconsistency and the idea that such a demonstration could be considered as a "theory of involuntary unemployment". For Neisser, an inconsistent system was simply wrong hence vacuous: "Inconsistency indicates that something is wrong with the system, but it is not a guide to what is wrong" (H. Neisser, Some Notes on Don Patinkin's on the Consistency of Economic Models, 17 July 1947, 7).

Second, they rejected the general method he followed in order to define realized transactions. One could not define equilibrium quantities by way of a "compromise" between the equilibrium quantities of two heterogeneous models or Patinkin's "subsystems". An economist had to build a consistent model starting from agents' behaviors. As put by Koopmans:

The real world is not overdetermined. The task of the economist is not to work out a compromise between two logically incompatible theories, but to construct a realistic and consistent theory. This requires specifying what choices are left to each individual or group of individuals. (Koopmans, Comments on D. P. Theory of Unemployment, March 1947)

Third, they all stressed that his theory of compromise referred to a world of imperfect competition and was thus irrelevant in the context of a perfectly competitive system like the one he wanted to study. Besides, if imperfect competition was introduced, agents' maximization programs would have to be adapted accordingly which was not the case in Patinkin's thesis. Koopmans, Marschak and Simon all insisted on this point:

"2. Is not the theory of compromise and bargaining power beside the point? While bargaining power is a very important phenomenon of the real world, it does not exist in a perfectly competitive system. (...). And if

phenomena of imperfect competition like bargaining strength are introduced, this would require a modification of the equation of competitive behavior, rather than a compromise in the form of weighted average of the solutions of two incompatible competitive equation systems” (Marschak, Comments on D. P. Unemployment in Keynesian Systems, December 1946).

But it is not clear whether you think of workers and employers as individuals or as groups. In the latter case, the theory of imperfect competition would be needed to specify the choices open to each party and so “explain” unemployment” (Koopmans, Comments on D. P. Theory of Unemployment, March 1947)

The whole idea of bargaining power introduced here is mystical. How and why do they bargain? The only way to bargain in a competitive system is by changing your offer. (H. A. Simon, March 19, 1947, Memorandum, 3)

Finally, Koopmans and Simon pointed out the ad hoc character of the theory of compromise and the arbitrariness of the whole construction:

You introduce ‘institutional’ coefficients α , β , which are not further reduced to behavior patterns of individuals, as the essential determinants of unemployment. This is not a theory, but a definition. (Koopmans, Comments on D. P. Theory of Unemployment, March 1947)

There is an arbitrariness underlying the whole approach. (H. A. Simon, March 19, 1947, Memorandum, 1)

The critiques that the members of the Cowles Commission addressed to the thesis must have deeply shaken Patinkin. Yet, they failed to help him find his way out of the theory of compromise. The suggestion to elaborate upon imperfect competition theories was clearly at odds with Patinkin’s aim, for he wanted to show that unemployment could appear even in the context of a perfectly competitive system. Simon suggested that he developed his unemployment theory along dynamic lines. The model would have been a set of differential equations specifying how prices and wages changed when excess supply characterised both the goods and the labor market. But this did not answer the basic issue raised by the thesis. How could one account for the fact that agents were off their supply curves? Worst of all, Cowles Commissioners missed the basic flaw of Patinkin’s theory of unemployment. What he had presented as an equilibrium unemployment was not so. For as a result of the simplifications he had introduced, money prices and the rate of interest remained indeterminate. And as long as excess supplies were supposed to dominate the markets for goods and labor, wages and prices should fall. Only after the submission of the thesis did Patinkin understand the problem. The draft of a paper probably written during this period bears evidence of this realization. In this text, Patinkin still clung to the explanation of unemployment in terms of an “inconsistency”, but the presentation of his model was now accompanied by the following qualification:

First, this unemployment situation cannot be an equilibrium situation; for from (9.2) we see that the price level will keep on changing and will never settle down as long as there remains any unemployment. (Patinkin, *Inconsistent Systems and Involuntary Unemployment*, 1948?, 27)

Patinkin's thesis put forward a new research program. But its definition was still confused and the resulting theory very awkward. Cowles Commissioners were able to point out this awkwardness but they did not understand the consistency and the relevance of his undertaking behind the flaws.

4. Appraising the thesis: the concept of additional restraint in retrospect

Patinkin tried to elaborate a disequilibrium theory, a theory showing how agents could fail to complete their desired plans when markets did not clear, and this on the basis of a Walrasian model. In doing this, he identified the main modification necessary to turn a Walrasian model into a disequilibrium model: the modification of the Walrasian budget constraint. In Patinkin's language, agents in disequilibrium acted under "additional restraints". With hindsight, it appears that Patinkin and the members of the Cowles Commission missed the implications of this idea. We must try to understand why Patinkin could not put his finger on the key concepts of the disequilibrium theories of the 1970's, the "spillover effect", that he would elaborate in latter works, and Clower's "dual decision hypothesis", although all the elements for such a discovery seemed to be on the table. Why did he fail to give an operational content to his notion of choice under additional restraint? Analytical mistakes played an important role in this false start.

4.1. Firms

In *Money, Interest and Prices* (1956), Patinkin applied and developed an analysis of "intermarket pressures" introduced in a 1952 article about "The Limitations of Samuelson's 'Correspondence Principle'". As he put it in chapter 10 (1956, 157), an excess demand on one market could "spillover" on other markets and affect directly their prices. For when individuals could not buy all they want on one market, they would try to use their unspent funds to buy other commodities. Symmetrically, when firms could not sell all their production, they would limit their purchases and in particular their labor demand (1956, 217). This way, an excess supply on the goods market would "spillover" on the labor market. Clower (1965, 113) renamed Patinkin's "intermarket pressure" the "spillover effect". The expression was then used by Grossman (1971, 948), who tried to elaborate a "theory of

spillover” with “a choice theoretic basis”, and played a crucial role in the literature on disequilibrium that developed in the early 1970s.

A notable feature of Patinkin’s 1947 theory of unemployment is that, although his Keynesian macromodel did feature a spillover effect, he did not bring out this concept. His analysis contained the notion of voluntary exchange. Faced with a sales constraint on the goods market, producers adjusted their production to the demand for goods independently of the real wage. The level of output resulting from the short side adjustment on the goods market determined the employment level on the labor market. This employment level was expressed by equation $N_0 = \phi^l(Y_I)$ and appeared as a vertical line on figure 2. The excess supply on the goods market led to an excess supply on the labor market. Furthermore, involuntary unemployment appeared although the real wage was inferior, not superior, to the equilibrium real wage, a point on which Patinkin would insist again in chapter 13 of *Money, Interest and Prices* and that would impress Barro and Grossman (1971, 83)²⁴.

Now, even though the model featured a spillover effect, Patinkin failed to theorize it. The main reason for this was his lack of attention to the interaction between the goods market and the labor market. I must recall the reader that my presentation in section 2.2 gathered pieces of analysis that were presented in two different chapters of the dissertation. In chapter 7, Patinkin explained the determination of income on the goods market and in chapter 9 he explained the determination of the real wage on the labor market. So, whereas in chapter 13 of *Money, Interest and Prices* he centered the discussion on the interaction between the two markets, in 1947 he did not analyze explicitly this interaction.

In his dissertation, Patinkin could have even gone further than he did in *Money, Interest and Prices*. In his book, Patinkin would point out that the level of the aggregate demand X^D imposed a new constraint on firms’ choice and he would identify the vertical line N_0 as firms’ effective demand for labor well distinct from their notional demand (1956, 219). But at the same time, in a famous footnote (1956, footnote 9, 220), he presented this result as a puzzle without solution and did not go beyond his verbal exposition of the problem²⁵. In the thesis, on the basis of his representation of firms maximization program and of his analysis of the

²⁴ “The assumption of the previous section is that in times of unemployment, there is no a priori reason to expect the real wage to equal the marginal product. In the traditional theory, this equality is presumably brought about by the competition of employers to obtain workers; but in times of unemployment, such competition obviously does not exist” (Patinkin, 1947, 113).

²⁵ See Boianovsky (2006) for a detailed analysis of Patinkin’s hesitations on this point.

compromise on the market for goods, his attempt to formalize firms' choice under additional restraints could have led him to write the very program that was put forward by Barro and Grossman (1971, 85) as an interpretation of his 1956 contribution:

Maximize $pY - wN^D$ subject to $Y = \Phi(N)$ and $Y \leq Y_I$.

The result would have been an effective demand for labor of the form $N^{D'} = \Phi^I(Y_I)$ and a “choice theoretic basis” for the spillover effect.

But in chapters 7 and 9 dedicated to his macroeconomic models, Patinkin did not analyze agents' decision programs. And when he finally considered firms' decisions, in the “microeconomic” model of chapter 8, he considered a different and somewhat peculiar disequilibrium state. The additional restraint derived from the model bore on the demand for labor ($N^D \equiv \bar{N}$) and not on production. This led to a curious result. Firms expressed an effective supply of goods $Y^{S'} = F(\bar{N})$. And since \bar{N} was superior to their desired demand for labor N^f , this effective supply was larger than their desired supply²⁶.

Furthermore, Patinkin failed to see what he had achieved. Even though the definition of the additional restraint was inappropriate, firms' programs defined a function of the “ g_i ” type, an effective behavior function, well distinct from their Walrasian supply function. The additional restraint $N^D \equiv \bar{N}$ being given, firms chose their level of production. Yet, according to Patinkin, the “compromise” fixed the only decision variable of the agent (the capital stock being constant). For this reason he concluded: “There is no room to maximize. (...). Thus the values of these variables [Y and N^D] are uniquely determined apart from any profit maximizing by the firm” (1947, 94).

4.2. Households

In the case of households Patinkin had gathered all the ingredients necessary to find Clower's concept of “constrained demand functions” (1965, 119). Using his formalization of household maximization program (1947, 52-3) and following his definition of workers choice under additional restraint (1947, 94) we get the following result. As workers households maximized

$$U(WN/P, L)$$

subject to

²⁶ “Employers recognize that workers as a group desire to offer more labor than employers as a group desire to purchase. Therefore they will compromise and agree to hire more workers” (Patinkin, 1947: 93).

$$L + N = T \text{ and } N = \bar{N}$$

Therefore as consumers, they should have maximized

$$U(Z_2, \dots, Z_n)$$

subject to the budget constraint :

$$p_2 Z_2 + \dots + p_n Z_n = y \text{ where } y = W \bar{N} + \Pi^{27}$$

The result should have been a set of effective demands of the form $Z_i' = Z_i'(p_1, \dots, p_{n-1}, y)$. Since the Z_i were supposed to be homogeneous of degree zero with respect to the money prices and wN (1947, 54), they could have been written under the form $Z_i' = Z_i'(p_1/p, \dots, p_{n-1}/p, Y)$ where $Y = y/P$. Above all, Patinkin should have realized that these demands were distinct from the Walrasian demands resulting from the maximization program without the additional constraint, demands depending only on prices: $Z_i = Z_i(p_1, \dots, p_{n-1}, W)$. And since Patinkin assumed $\bar{N} < N^e$, then $Z_i' < Z_i$, where Z_i was the demand for good i determined by the “workers’ model”. Apparently, he could have shown that unemployment implied an effective demand for goods inferior to the notional demand and dependant on real income as an exogenous variable like Clower did in 1962 at the Royaumont conference.

Yet, Patinkin missed the point. All along the elaboration of his unemployment theory he considered that the Keynesian demand function X^D could be derived from the standard Walrasian maximization program. In contrast with Clower, he did not see the presence of income among the arguments of the consumption function as an analytical problem to be explained outside Walrasian theory. As I showed above, this mistake was written down in the most explicit way at the beginning of chapter 8 (1947, 89). A similar point was made in chapter 7 when Patinkin presented his modified diagonal cross diagram:

For sections 38 and 39, we note that the expenditure function $X^D = X^D(Y)$ represents the actions of buyers when they are free to fulfill their desires; i.e. their actions subject only to the restraint of the technological functions. (1947, 82)

²⁷ Here I preserve Patinkin’s presentation which did not distinguish clearly between goods, bonds and money. Note in particular the absence of any real balance effect. Furthermore, at various stage of his presentation, Patinkin assumed two propensities to consume, one out of wages and the other out of profits. And in chapter 6, profits did not appear in the maximization program of “individuals”. In others words, he introduced implicitly a class structure in his economy. In order to simplify the presentation, I assume that workers receive firms’ nominal profits Π .

Starting from this premise, Patinkin considered that he did not have to examine the choice of consumer-workers in the context of unemployment.

If we go back to the section 28 of the thesis, mentioned in the quotation from chapter 8, we can see how the mistake crept in. Patinkin explicitly raised the issue of the introduction of income in the demand functions:

The discussion in this chapter will concern itself with macroeconomic models. Usually a fundamental role in these models is played by the variable y representing money income. But our previous analysis has developed the excess demand functions as dependent on prices only. Our first task will accordingly be the investigation of changes in the maximization procedure which will cause the excess demand functions to depend on y also. For this to be true, y must bear the same relationship to the individual as did prices in the Walras-Pareto system. In other words, when the individual comes to decide on his purchases, he must consider his income as a given quantity which he can no longer affect. And this given income must then be allocated among the different goods. (1947, 52)

Patinkin believed that a “double maximization procedure” was the solution²⁸. The choice of workers between leisure and labor would determine consumers’ income. As a consequence, income could be treated as an exogenous datum of the choice between different goods (as in the maximization program presented above).²⁹

With hindsight, this approach and the confusion behind it are difficult to understand. Since Patinkin wrote down the maximization program underlying workers’ labor supply, he should have realized that their income was an endogenous variable dependant on the real wage. The choice theoretic basis of his macroeconomic models implied that the income variable entering aggregate demand could not vary unless the real wage itself varied. And this contradicted his analysis of the goods market in chapters 7, 8 and 9. But Patinkin followed the treatment of this issue found in the works of Klein and Lange.

²⁸ “One way of doing this is to consider the maximization procedure as broken up into two steps.” (1947, 52)

²⁹ Another paragraph of the thesis shows how close Patinkin got to Clower’s effective demand theory: “Secondly, in order to introduce y_a into the demand functions it is not necessary to assume a double maximization procedure of the type discussed here. All that is necessary is that in some way the individual should consider his money income given when he comes to select his purchases. Thus in the case where workers have no choice over either working hours or wage rate, their income will be taken as given and will enter their demand functions.” (1947, 53-4) Patinkin’s hesitation concerning the variable that cannot be decided by workers, “working hours or wage rate”, shows again that his thought was still confused on this matter.

In the “technical appendix” of *The Keynesian Revolution*, Klein (1947, 258-61)³⁰ explained the “Mathematical derivation of the system of the General Theory”. The basis for the derivation of the Keynesian consumption function, the demand for bonds and the liquidity preference function was a maximization program in which income appeared as an exogenous variable. Though, in his analysis of the message of the *General Theory*, Klein assumed that the labor supply was an increasing function of the real wage (1947, 74-5 and 87), like Patinkin, he dissociated the choice between labor and leisure and the decision concerning the allocation of income. Unlike him, he did not comment on this option and the foundations of the labor supply were not explained. Klein simply followed the standard procedure:

In the theory of consumer behavior, the usual practice is to start out with the utility function (or a monotonic transformation of the same), derive the utility-maximization equations, and finally solve the demand equations in terms of prices and income. (Klein, 1946, 306)

In *Price, Flexibility and Employment*, Lange wrote down the choice of a consumer only once and in this instance he also treated income as an exogenous variable (1944, note 6, 16). And in “The Rate of Interest and the Optimum Propensity to Consume”, he argued that income was the hidden adjusting variable of the Walrasian system:

By introducing the prices of all commodities [Walras] brings income indirectly into the equation expressing the propensity to save (1938, 21-22).

The “maximization under additional restraints” was the solution of a puzzle that would wait for an adequate formulation until 1962 and Clower’s contribution to the Royaumont Conference. It is fascinating to see how, despite coming so close to Clowers’ breakthrough, Patinkin was not able to grasp the issue at hand and find the solution laying before him in his own systems of equations. He realized that the compatibility between the Keynesian aggregate demand function and general equilibrium price theory had to be clarified somehow. His own formulation of the maximization program of the worker-consumer in a situation of involuntary unemployment implied Clower’s notion of “constrained demand functions”. But he did not understand how deep the issue was. He could not escape from the approach followed by his contemporaries simply assuming that income was an exogenous variable of consumers’ choice in the Walrasian setting. Patinkin considered his additional restraint concept as an answer to another question. Writing down the programs of firms and

³⁰ The reference is to the 1966 edition of *The Keynesian Revolution*, but this passage was already in the 1947 edition (1947, 192-5).

households in the context of unemployment equilibrium was necessary in order to offer “a precise mathematical definition of ‘involuntariness’ or coercion corresponding to our intuitive ideas” (1947: 105). More was a stake than a question of definition. But he did not see that modelling choices under additional restraint could help understand the interactions between different markets and the foundations of a non-Walrasian equilibrium.

4.3. The remainders of a Marshallian training

As we noted in section 1.1, Patinkin’s basic training was Marshallian. In 1947, his command of Walrasian general equilibrium theory was still in its infancy. This throws some light on the shortcomings of his thesis.

The fact that Patinkin never tried to analyze the interaction between the goods market and the labor market was an important source of confusion. Although he dealt with a general equilibrium system and referred to the Lausanne school, his approach was mainly partial equilibrium. The organization of Patinkin’s presentation illustrates the issue. In each chapter he considered one market only. In chapter 7 he analyzed the compromise on the market for goods. In chapter 8 he discussed the compromise over employment on the labor market. And in chapter 9 he discussed the compromise over real wage on the labor market. This is typical of a period in which economists began to deal with general equilibrium system with a Marshallian training³¹.

The sequential conception of consumers’ maximizing program presented in the preceding section is another characteristic of the Marshallian approach (De Vroey, 1999a, 333). In a Walrasian general equilibrium framework, agents are supposed to determine simultaneously their offers and their demands for all commodities including labor. In this context, income is a dependent variable of consumers’ decisions in contradiction with the Keynesian consumption function and the Keynesian liquidity preference function. But economists like Klein, Lange or Patinkin were not fully fledged Walrasian. They viewed consumers’ choice as a two step process and either thought that this approach resolved the apparent contradiction, like Patinkin, or remained totally unaware of the difficulty.

As stressed by De Vroey (1999a, 322), a hallmark of the Marshallian approach is the assumption that agents try to guess the equilibrium price of the market before it opens. As in Marshall’s corn market (chapter 2, book V of the *Principles*), agents anticipate the demand

³¹ For the lack of adequate method to deal with general equilibrium in Modigliani (1944), see Rubin (2004).

curve and the supply curve of a given market, deduce the equilibrium price and decide the quantity that they will produce or demand accordingly. Even though Patinkin made no reference to expectations, the approach he followed in chapter 8 of the thesis was of a Marshallian sort. As I explained in section 2.3, firms and workers determined their desired levels of employment on the basis of their own models of the economy. But of course, in Patinkin's theory agents could not guess the correct magnitudes.

5. Conclusion: Patinkin's thesis and later disequilibrium theories

And it is this personal experience of knowing, but not knowing –knowing something, but not realizing its “obvious” implications for other problems with which I was concurrently dealing until a later point of time, an experience that I have had on other occasions as well –that has strongly influenced my subsequent work in the history of doctrines, especially that dealing with the discovery of the “General Theory”. (Patinkin, 1995, 382)

In spite of all the confusion surrounding Patinkin's work, his Ph.D. thesis contained in embryo the research programs of disequilibrium economists of the 1970's. I use the plural for, as Backhouse and Boianovsky (2005, 2006) have shown, the research programs pursued by disequilibrium theorists were heterogeneous. Patinkin's work contained the basic issues and the seeds of the concepts that would feed later developments. Some of these ideas were elaborated by Patinkin himself, others would have to be reinvented. For there was still a long way from his 1947 theory of unemployment to the works of disequilibrium theorists in the 1970s. The thesis was an instance of “knowing, but not knowing”.

As we showed in part 1 and 2 of the paper, Patinkin attempted to reconstruct Keynes' theory of involuntary unemployment taking the Walrasian model as his based camp. From this perspective, the problem raised by the Keynesian theory became the analysis of transactions taking place in disequilibrium or at prices that were not compatible with market-clearing. This research program was characteristic of the contributions of Clower (1965) and Barro and Grossman (1971) and even more obvious as far as the French school of Benassy (1975) or Grandmont (1977) is concerned (De Vroey, 2004, 117). This project led Patinkin to lay down a number of concepts that pointed in the direction of disequilibrium theories. The theory of compromise showed that in a disequilibrium context, an adjustment mechanism distinct from the price mechanism was needed. It also implied the notion of a non-Walrasian equilibrium. In contrast with the stance of his later contributions (Patinkin, 1948 and 1956), in 1947 Patinkin intended explicitly to account for the possibility of an unemployment equilibrium. Here history moved in spiral. Non-Walrasian equilibria were explored in the

1970s but economists based their analysis on the fixed price assumption that Patinkin always rejected. In 1947, Patinkin showed clearly that, in a disequilibrium context, agents were faced with additional constraints so that the choice theoretic basis of the general equilibrium models had to be modified. Furthermore, he realized that these additional constraints would generate demand functions (“ g_i ”) distinct from the Walrasian demand functions (“ h_i ”). In the case of unemployed workers, he identified correctly the additional constraint as the level of employment and came very close to Clower’s 1965 breakthrough. Besides, Patinkin wrote down the very diagrams that he would use in 1956 to analyze the behavior of firms faced with a sales constraint. The models of the thesis already contained an implicit spillover effect. In other words, the thesis contained the main ingredients needed to elaborate the choice theoretic basis of spillover effects and obtain a first understanding of interactions between markets in disequilibrium.

But the conceptual distance between the thesis and later works was still wide. Eventually, Patinkin was not able to bring out the spillover effect or Clower’s notion of constrained consumer demand function. He had the general intuition but he could not see how to give it an operational content. Patinkin probably realized that his intuition could have led to something deeper. This is why in his conclusion he came back to the notion of additional restraint stating that a result more general than the behavior of firms and workers presented in his chapter 8 was possible:

62. In the analysis of chapter VIII it turned out that when the additional restriction was used, the behavior unit was no longer free to maximize utility or profit. In general, this need not be so; maximization subject to the additional restriction may still result in a schedule of alternatives. (1947, 116)

But Patinkin’s reasoning was impeded by a partial equilibrium approach. In addition, the standard theory of consumers’ choice taking income as an independent variable prevented him from realizing the breach of continuity between the behavior of the Keynesian consumer and the behavior of the Walrasian consumer.³² Actually this problem remained the blind spot of his analysis in chapter 13 of *Money, Interest and Prices*. And even when he discussed Clower’s article on the “Keynesian counterrevolution” at the Royaumont conference in 1962, he did not realize the relation between Clower’s “dual decision hypothesis” and his chapter

³² This explanation was already hinted at by Stanley Fisher (1993: 21) in relation to *Money, Interest and Prices*, a fact that Mauro Boianovsky brought to the attention of the author.

13.³³ Last but not least, the theory of compromise was a very ad hoc construction and, as Patinkin soon realized, it left the price level and the rate of interest indeterminate. What he had considered as a theory of unemployment equilibrium was actually a theory of unemployment disequilibrium in the sense that prices were moving.

For all these reasons, Patinkin's Ph.D. dissertation should be considered as the pre-history of disequilibrium theories. History began with chapter 13 of *Money, Interest and Prices*, the result of a long and tortuous process of elaboration.³⁴ In this chapter, Patinkin applied the concept of spillover in a Keynesian context and finally captured the operational intuition he had missed in his thesis. Faced with a sales constraint on the market for goods, firms would have to revise their demand for labor without taking into account the level of the real wage. This in turn would create an excess supply of labor and involuntary unemployment. The comparison between the thesis and chapter 13 clarifies the nature of Patinkin's 1956 breakthrough. First, he had passed from a partial equilibrium to a general equilibrium approach. The two key figures of the thesis (figure 1 and figure 2 above) survived in chapter 13. But whereas in 1947 Patinkin analyzed each figure separately, in 1956 he focused on the interaction between them. The novelty was the tâtonnement method inspired by Lange (1944) and Walras (1926) and developed in the preceding chapters of the book.³⁵ Second, Patinkin no longer left the wage rate, the price level and the rate of interest hanging in the air. He considered different scenarios on the basis of the relative adjustment speeds of prices. This approach was far from conclusive (De Vroey, 2002) but it had the merit to close the gap. Last, Patinkin argued more forcefully than he did in the thesis that the behavior of firms in the Keynesian scenario was at odds with the Walrasian behavior. This chapter 13 was a decisive

³³ "First, although I was a discussant of Clower's well-known article "The Keynesian counterrevolution": a theoretical appraisal" (1965) when it was first presented at a 1962 conference, only some time after I had published the second edition of my *Money, Interest and Prices* (1965) (on which I was working at the time of the conference) did I realize that his "dual decision hypothesis" about the behavior of consumers who were recipients of income from labor was simply the obverse side of the contention of my chapter 13 that the inability of firms to sell the quantity indicated by their supply curve for output, drawn at a given real wage, would make unwilling to employ the amount designated on their demand curve for labor for that real wage." (Patinkin, 1995, 382) This anecdote is confirmed by the reading of the discussions printed in the volume edited by Hahn and Brechling (1965) after the Royaumont conference.

³⁴ On this process see Boianovsky (2006).

³⁵ On this point see Rubin (2004, 198).

source of inspiration for Clower (1965) (Patinkin, 1989, xvii; Rubin, 2005b) and Barro and Grossman (1971) after him.

Yet, in some respects, Patinkin's thesis resembled disequilibrium theories of the 1970s more than his chapter 13. Though he still used the "additional restraint" in his definition of involuntary unemployment (1956, 212), the concept was totally absent from the rest of chapter 13. In *Money, Interest and Prices*, Patinkin returned to the "old" style of Marshall and Hicks" (Backhouse, 2002, 188) giving priority to verbal reasoning over algebra. This was probably a consequence of the thesis experience. In his autobiography, Patinkin regretted the emphasis on mathematics in his dissertation, and explained how afterwards he had tried to give a priority to the economic intuition:

For like most doctoral students (then, and I am afraid even more so now), I attributed too much importance to the technique and formal mathematical analysis. And so my thesis gave much emphasis on the rigorous derivation of theorems from definitions, assumptions, and preliminary lemmas, while devoting inadequate attention to the economic interpretation of the analysis. (1995, 383)

We have seen how Patinkin's burning enthusiasm for mathematical economics led to the burgeoning models of the thesis. Yet the results were disappointing for him. The artificiality and the arbitrariness of his complex apparatus was severely criticized by his mentors at the Cowles Commission and he finally realized that what he had presented as an equilibrium was a disequilibrium state. Like the Keynes' of the *Treatise on Money*, as Patinkin would write years later (1984, 97-8) quoting the *General Theory*, he had lost himself in a "maze of pretentious and unhelpful symbols". This partially explains why he did not formalize the analysis of chapter 13 in 1956. The absence of a mathematical appendix has probably led many readers, like Leijonhufvud, to think that chapter 13 was more superficial than the others, an amateur incursion into the field of Keynesian theory. The opposite was true. Knowing by experience how difficult the mathematical translation was, Patinkin consciously concentrated on the essential, the economic intuition. As he put it in a letter to Leijonhufvud in 1974, he "did not succeed in achieving an integration of [his] economic intuition with [his] formal analysis". As shown by Boianovsky's presentation of the correspondence between Nissan Liviatan and Patinkin, Patinkin came up against the same objection over and over again. Like the members of the Cowles Commission before him, Liviatan asserted that Patinkin's analysis of the process of income determination implied imperfect competition³⁶. As a consequence of these criticisms, Patinkin abandoned the ambition of the thesis. He put forward what he

³⁶ See Boianovsky (2006, 231-232).

considered to be his most important intuition, the spillover effect, and kept assuming perfect competition but he abandoned the idea of formalizing involuntary action as the outcome of choice under additional restraint. Yet, we suspect that a more profound change of mind was at work behind the exclusion of the concept of additional restraint from Patinkin's central message. In his thesis, Patinkin reconstructed agents' maximizing programs along the lines that would be followed by Barro and Grossman (1971) and this without remorse. The correspondence with Liviatan and Patinkin's comments on Barro and Grossman (1971) at the Perugia conference (Boianovsky, 2006, 244) seem to indicate that at some point he became convinced that the approach of the thesis was faulty. In 1956 he had come to believe that his programs with additional constraints were inconsistent with the standard behavior of agents in a perfectly competitive system. This way of rationalizing and formalizing the spillover effect was not acceptable. The solution that he called for, in note 9 of chapter 13, was not the elaboration of a non-Walrasian micro like in the thesis but a more sophisticated Walrasian micro. In a letter to Liviatan about the behavior of firms in chapter 13, Patinkin wrote : "I am interested in working on the traditional acceptable way of price theory" (quoted by Boianovsky, 2006, 232). The same position appeared in the new introduction of the second edition of *Money, Interest and Prices* in 1989 where he stressed the fact that disequilibrium macroeconomics "violates the assumption of rational economic behaviour" and quoted with approval the works of New Keynesian that "rationalized the seemingly irrational" (1989, xviii). To conclude, chapter 13 marked a crucial step forward in the elaboration of disequilibrium macroeconomics but it may have contained simultaneously a step backward announcing Lucas notion of equilibrium "discipline". What Clower (1965) or Barro and Grossman (1971) did was to reinvent the concept of choice under additional restraint that Patinkin had put aside along the way.

Appendix

Maximization under "additional restraints": a general formulation

"For instance, consider a consumer maximizing his utility from n goods

$$(62.1) \quad u(Z_1, \dots, Z_n)$$

Subject to the budget restraint

$$(62.2) \quad p_1 Z_1 + p_2 Z_2 + \dots + p_n Z_n = y$$

(where p_i = price and y = income are considered as given) and some additional restriction (say, some sort of rationing control)

$$(62.3) \quad \theta(Z_1, Z_2, \dots, Z_n) = 0.$$

Then he maximizes

$$(62.4) \quad u(Z_1, \dots, Z_n) - \lambda_1 (p_1 Z_1 + p_2 Z_2 + \dots + p_n Z_n - y) - \lambda_2 \theta(Z_1, Z_2, \dots, Z_n)$$

To yield the n equations

$$(62.5) \quad u_i - \lambda_1 p_i - \lambda_2 \theta_i = 0 \quad (i = 1, \dots, n).$$

Thus we have $n+2$ equations (62.2), (62.3), (62.5) in $n+2$ variables: the Z_i , λ_1 , and λ_2 . Solving in terms of the p_i and λ we have

$$(62.6) \quad \lambda_j = f_j(p_1, \dots, p_{n-1}, y) \quad (j = 1, 2)$$

$$(62.7) \quad z_i = g_i(p_1, \dots, p_{n-1}, y) \quad (i = 1, \dots, n)."$$

(1947a : 116-117).

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Abstract: In the opening sentence of *Money, Interest, and Prices*, Patinkin noted that his book was the outgrowth of ideas first presented in his doctoral dissertation. This claim has attracted the attention of most scholars who wrote about his works in recent years. As shown by Boianovsky (2006), Merhling (2002) or Rubin (2002), reading Patinkin's doctoral dissertation shed new light on his major work. However, these articles only contain partial presentations of the thesis. This essay contributes to fill in this gap. It offers a detailed presentation of the second part of Patinkin's Ph.D and claims that this document foreshadowed the research programs of disequilibrium theorists of the 1970's.

Keywords: Patinkin, disequilibrium theory, microfoundations, Keynesian theory.